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AN IMPROVED FEEDBACK LOOP FOR ADAPTIVE ARRAYS.(U)

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This report is concerned with the problem of time constants in adaptive arrays. The report presents an improved form of adaptive array feedback loop, which has the property that its time constants are fixed. This property is an advantage over the well-known LMS loop, for which time constants depend on received signal power. Fixed time constants are of interest because they simplify dynamic range problems for adaptive arrays in communication and radar systems.

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CONTENTS

I. INTRODUCTION

Adaptive arrays based on the LMS algorithm (1) are very appealing as a means of protecting communication systems from interference (2,3). These antennas can automatically track desired signals and simultaneously null interfering signals. Moreover, they can do this using conformally mounted elements on an irregular surface, such as an aircraft. Possible applications of adaptive arrays include communication systems subject to accidental interference, RFI and jamming.

An important problem with adaptive arrays, however, is their limited dynamic range. Two factors restrict the dynamic range of an adaptive array. First, there are equipment limitations. The multipliers, amplifiers, etc., in the LMS feedback loops operate properly only over a certain range of signal power. However, although equipment limitations are very real, they are not fundamental; that is, they are not due to the LMS feedback concept itself. Rather, they appear because the equipment performance does not match the mathematical behavior of the LMS algorithm. Also, it is possible to overcome such limitations with improved design.

The second factor limiting dynamic range is more fundamental, because it is inherent in the LMS algorithm. The limitation occurs because the speed of response of the LMS feedback loop depends on received signal power. The array responds slowly to a weak signal and rapidly to a strong signal. This situation makes it difficult to accomodate a wide range of signals because, in most applications of adaptive arrays, system requirements limit both the minimum and maximum speed of response of the array. As a result, the array can handle only a limited range of signal power without exceeding speed of response bounds.

In this report we address the problem of variable time constants in the LMS algorithm. The purpose of the report is to present an improved form of adaptive array feedback loop that appears to solve this problem. The feedback loop described produces the same steady-state weights as the LMS algorithm, but has the property that its time constants are nearly independent of signal power.

In Section II of the report, we establish notation and discuss the time constant behavior of the LMS array. In Section III, we determine an "ideal" control law for the adaptive array. In Section IV, we describe a feedback loop whose performance approximates this ideal control law. Section V presents an example showing the transient behavior of an array using this feedback loop.

II. DYNAMIC RANGE LIMITATIONS IN THE LMS ARRAY

An LMS adaptive array (1) has weights controlled by the gradient law

$$\frac{dw_i}{dt} = -k\nabla_{w_i}[\epsilon^2(t)] , \quad 1 \leq i \leq 2M \tag{1}$$

where w_i is the i^{th} array weight, k is a positive constant, M is the number of elements in the array and $\nabla_{w_i}[\epsilon^2(t)]$ is the i^{th} component of the gradient of the squared error signal. The error signal $\epsilon(t)$ is defined by

$$\epsilon(t) = R(t) - \sum_{j=1}^{2M} w_j x_j(t), \tag{2}$$

where $R(t)$ is the Reference Signal (or "Desired Response" (1)) and $x_i(t)$ is the i^{th} quadrature signal in the array. Substituting Equation (2) in Equation (1) yields

$$\frac{dw_i}{dt} = 2k\, x_i(t)\, \epsilon(t), \tag{3}$$

which corresponds to the feedback loop shown in Figure 1.

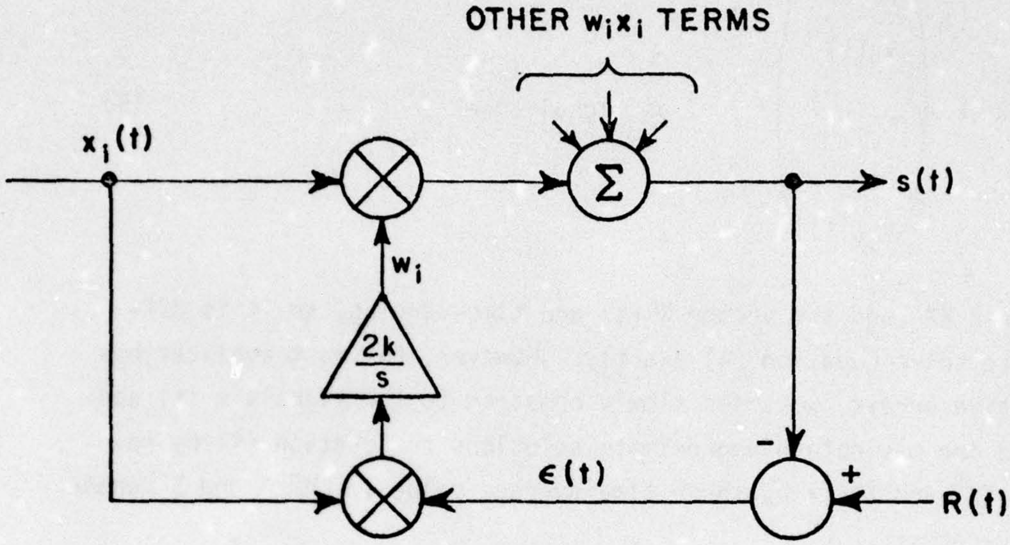

Figure 1. The LMS feedback loop.

By substituting Equation (2) into Equation (3) and collecting terms involving w_i on the left, we find that the weights in an LMS array satisfy the system of differential equations

$$\frac{dw}{dt} + 2k\ XX^T w = 2k\ XR(t), \tag{4}$$

where w is the weight vector,

$$w = \begin{pmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_{2M} \end{pmatrix} \tag{5}$$

and X is the signal vector,

$$X = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_{2M}(t) \end{pmatrix} \quad . \tag{6}$$

The matrix XX^T and the vector $XR(t)$ are time-varying, so it is difficult to solve Equation (4) exactly. However, for most applications of adaptive arrays, w varies slowly compared to the signals $x_i(t)$ and $R(t)$ and one may obtain approximate solutions to Equation (4) by replacing XX^T and $XR(t)$ by their time average values. Let Φ and S denote these averages:

$$\Phi = \overline{XX^T}, \tag{7a}$$

$$S = \overline{XR(t)} \; . \tag{7b}$$

With this substitution, the system in Equation (4) becomes

$$\frac{dw}{dt} + 2K\Phi w = 2kS, \tag{8}$$

which has constant coefficients and may be solved by diagonalizing Φ. The solution for the i^{th} weight is

$$w_i = A_{i_1} e^{-2k\lambda_1 t} + A_{i_2} e^{-2k\lambda_2 t} + \ldots + A_{i_{2M}} e^{-2k\lambda_{2M} t} + c_i, \quad (9)$$

(9)

where $A_{i_1}, A_{i_2}, \ldots, A_{i_{2M}}$ are constants determined by initial conditions, $\lambda_1, \lambda_2, \ldots \lambda_{2M}$ are the eigenvalues of Φ, and c_i is the steady-state value of w_i. We note that the time constant of the j^{th} exponential term is

$$\tau_j = \frac{1}{2k\lambda_j} \; . \tag{10}$$

A dynamic range problem arises in the LMS array because the eigenvalues of Φ, and hence the time constants, depend on the signal power received by the array. A strong signal produces a large eigenvalue and a weak signal produces a small eigenvalue. In a typical design problem the strongest signal is interference and the weakest signal is thermal noise. In this case it can be shown (4) that the maximum and minimum eigenvalues, λ_{MAX} and λ_{MIN}, are related by*

$$\frac{\lambda_{MAX}}{\lambda_{MIN}} \cong M \frac{I}{N} , \tag{11}$$

where I is the interference power and N is the thermal noise power. The eigenvalue ratio in Equation (11) can easily be quite large. For example, if we wish to null interference 60 dB above thermal noise, λ_{MAX} will be approximately 10^6 times λ_{MIN}. Equation (9) shows that a typical weight transient will include a fast exponential term associated with λ_{MAX} and a slow exponential term associated with λ_{MIN}. The time constants for these two terms will be in the ratio given by Equation (11). We will refer to this ratio as "time constant spread".

In most applications, there are design bounds on both the minimum and maximum time constants. For example, suppose an array is to operate in an aircraft communication system. On the one hand, the fastest speed of response is limited by the signal modulation rate. (If the weights are too fast, they interact with the desired signal modulation.) On the other hand, the slowest speed of response is limited by the need for the array to be fast enough to track aircraft motion.

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*The approximation is accurate as long as the interference has much greater power than any other signal present.

Thus, system requirements usually limit the allowable range of $\lambda_{MAX}/\lambda_{MIN}$. Because of Equation (11), there is then a maximum value of I/N that the system can accommodate. In the author's experience, this value is usually disappointingly low. One is often forced to accept a design that does not meet speed of response requirements in order to handle even a modest value of I/N. This is the dynamic range problem in the LMS array.

III. THE IDEAL CONTROL LAW

To find a way to correct this problem, it is helpful if we first examine the reason for time constant spread in a more basic way. We will observe that time constant spread occurs because the LMS algorithm is based on a gradient approach. A gradient approach causes the weight vector to move along some parts of its trajectory at a different speed than along other parts. This viewpoint will also suggest a way of modifying the LMS loop to solve the problem.

We may use a simple two-dimensional example as an illustration. From Equation (2), the average squared error may be written

$$\overline{\varepsilon^2(t)} = \overline{R^2(t)} - 2w^T S + w^T \Phi w \tag{12a}$$

$$= \overline{\varepsilon^2_{min}} + (w-w_{opt})^T \Phi (w-w_{opt}), \tag{12b}$$

where

$$w_{opt} = \Phi^{-1} S \tag{13}$$

is the weight vector yielding minimum $\overline{\varepsilon^2(t)}$, and

$$\overline{\varepsilon^2_{min}} = \overline{R^2(t)} - S^T \Phi^{-1} S \tag{14}$$

is the value of $\overline{\varepsilon^2(t)}$ when $w = w_{opt}$. (T denotes the transpose.) Consider a two-dimensional $\overline{\varepsilon^2(t)}$-surface defined by

$$w_{opt} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{15}$$

and

$$\Phi = \begin{pmatrix} 1 & 0 \\ 0 & 100 \end{pmatrix} . \tag{16}$$

The resulting $\overline{\varepsilon^2(t)}$-surface has elliptical loci of constant $\overline{\varepsilon^2(t)}$ in the w-plane, as shown in Figure 2. A typical trajectory traveled by a weight vector under the LMS algorithm is shown in Figure 2. The weight vector starts at an arbitrary point w_0 and travels to w_{opt} along the curved path shown. At each point of the trajectory, the weights move in the steepest-descent direction, which is always perpendicular to a constant $\overline{\varepsilon^2(t)}$ locus. Since the eigenvalues are unequal, the weights do not move in a straight line toward w_{opt}, but along a curved path, as shown. Moreover, because the LMS algorithm makes the time rate change of w proportional to the slope of the $\overline{\varepsilon^2(t)}$-surface, the weight vector moves from w_0 to w_1 in Figure 2 rapidly, since the slope is large in this region, but from point w_1 to w_{opt} slowly, because the slope is small in this region. The movement from w_0 to w_1 contributes a fast term to the weight response, and the movement from w_1 to w_{opt} contributes a slow term.

We observe that the spread in time constants would be eliminated if the weights were always forced to move in a straight line toward w_{opt} with a value of dw/dt not dependent on the slope of the surface. Such a preferred trajectory is shown in Figure 2. This preferred trajectory is in the vector direction $-(w-w_{opt})$, for any given w. However, the gradient of $\overline{\varepsilon^2(t)}$, as computed from Equation (12b), is

$$\nabla_w [\overline{\epsilon^2(t)}] = 2\Phi\,(w-w_{opt}), \tag{17}$$

so the LMS algorithm is

$$\frac{dw}{dt} = -2k\Phi\,(w-w_{opt}). \tag{18}$$

I.e., the LMS algorithm moves the weights in the direction of the vector $-\Phi(w-w_{opt})$, not in the direction $-(w-w_{opt})$. Since Φ has unequal eigenvalues, $-\Phi(w-w_{opt})$ is usually in a different direction than $-(w-w_{opt})$. Moreover, the presence of Φ in Equation (18) causes the slope of the $\epsilon^2(t)$-surface to influence the speed of response.

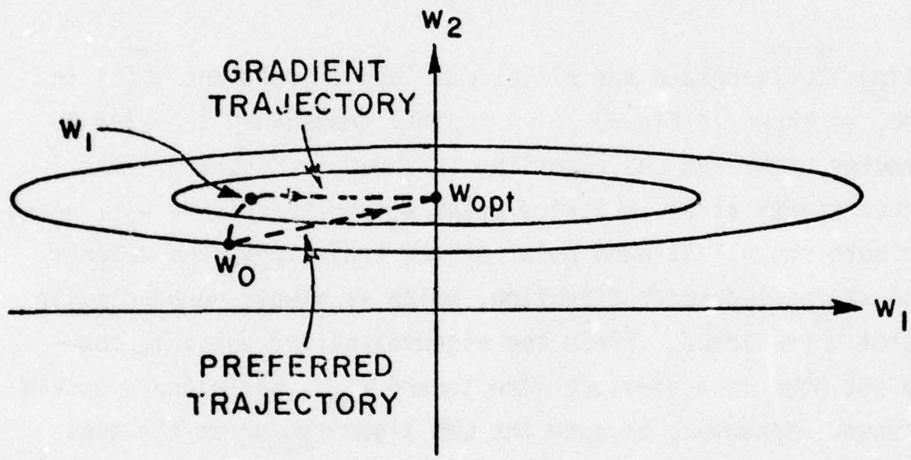

Figure 2. Constant $\overline{\epsilon^2}$ in the w-plane.

Clearly, a better strategy would be to eliminate the Φ from the right hand side of Equation (18), i.e., to control the weights according to the equation

$$\frac{dw}{dt} = -2k(w-w_{opt}). \tag{19}$$

If this were done, the weights would move directly toward w_{opt} and the eigenvalues of Φ would not influence the speed. To see how such a control law may be realized, we note from Equation (17) that

$$w - w_{opt} = \frac{1}{2} \Phi^{-1} \nabla_w [\overline{\varepsilon^2(t)}] . \tag{20}$$

Hence Equation (19) is the same as

$$\frac{dw}{dt} = -k \Phi^{-1} \nabla_w [\overline{\varepsilon^2(t)}] , \tag{21}$$

or equivalently,

$$\Phi \frac{dw}{dt} = -k \nabla_w [\overline{\varepsilon^2(t)}] . \tag{22}$$

We will refer to this as the <u>ideal control law</u> for the adaptive array. It differs from the LMS algorithm only in the presence of the matrix Φ multiplying dw/dt. In the next section, we describe a feedback loop that implements this equation.

IV. A MODIFIED FEEDBACK LOOP

When Equation (2) is substituted in Equation (3), the LMS algorithm becomes

$$\frac{dw_i}{dt} = 2kx_i(t) \left[R(t) - \sum_{j=1}^{2M} x_j(t) w_j \right] . \tag{23}$$

Let us consider, instead of this, the control law

$$\frac{dw_i}{dt} = 2k\, A \left\{ x_i(t) \left[R(t) - \sum_{j=1}^{2M} x_j - c \sum_{j=1}^{2M} x_j(t) \frac{dw_j}{dt} \right] \right\} . \tag{24}$$

In this equation, c is a gain constant and $A\{\cdot\}$ represents an averaging operation to be defined below. This equation corresponds to the feedback loop shown in Figure 3. This loop is similar to that in Figure 1, except for the inclusion of the averaging operation $A\{\cdot\}$ and the extra amplifier, summer and multipliers to form

$$c \sum_{j=1}^{2M} x_j(t) \frac{dw_j}{dt}$$

and subtract it from the reference signal.

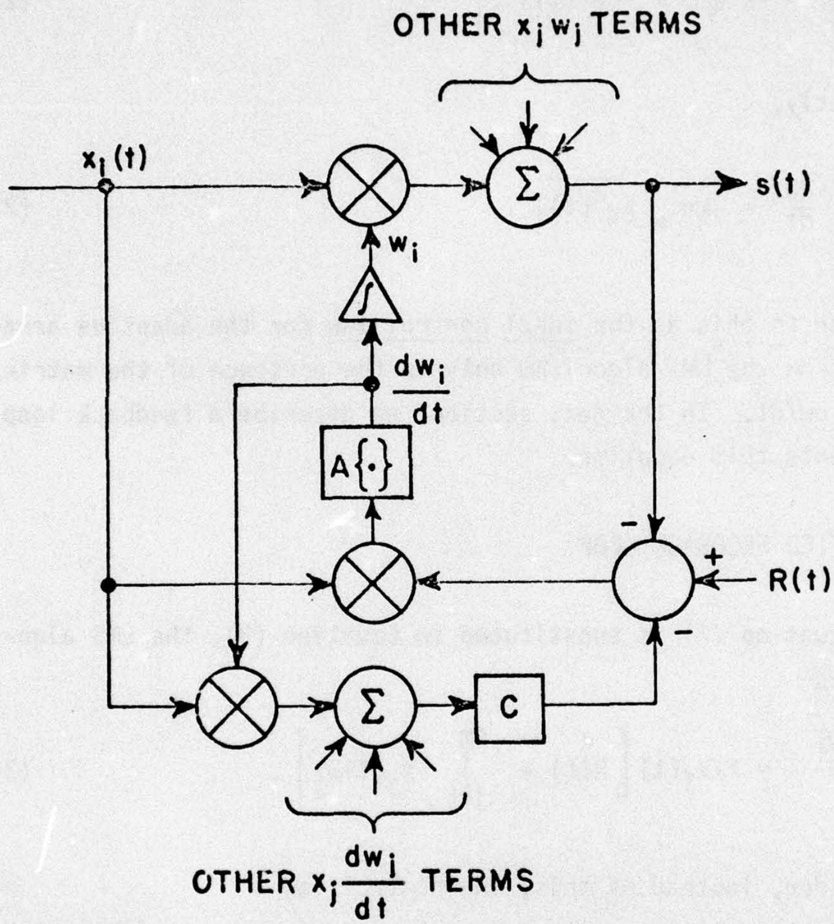

Figure 3. The modified feedback loop.

We define the averaging operation A{·} as a finite time average. We assume the weights are slowly varying in comparison to the signals $x_i(t)$ and R(t). We let A{·} represent a time average over an interval short enough that w_i and dw_i/dt may be considered constant over this interval, but long compared to the carrier cycles of $x_i(t)$ and R(t).

The actual value of the time interval to be used and the reasons for including this averaging in the loop will be discussed below. For the moment, we settle for this definition, which allows us to make the approximation

$$A\left\{x_i(t) \sum_{j=1}^{2M} x_j(t) \frac{dw_j}{dt}\right\} = \sum_{j=1}^{2M} A\{x_i(t)\, x_j(t)\} \frac{dw_j}{dt} \quad . \tag{25}$$

Since the right side of Equation (23) is simply $-k\, \nabla_w[\epsilon^2(t)]$, we may also write

$$2k\, A\left\{x_i(t) \left[R(t) - \sum_{j=1}^{2M} x_j(t) w_j\right]\right\} = -k\nabla_{w_i} [A\, \{\epsilon^2(t)\}] \quad . \tag{26}$$

Equation (24) may then be expressed in matrix form as

$$[I + 2kcA\, \{XX^T\}]\, \frac{dW}{dt} = -k\nabla_W\, [A\, \{\epsilon^2(t)\}]\, . \tag{27}$$

To understand why the feedback loop in Figure 3 is useful, let us suppose for the moment that the averaging operation $A\{\cdot\}$ is good enough that

$$A\, \{XX^T\} = \Phi \tag{28}$$

and

$$A\, \{\epsilon^2(t)\} = \overline{\epsilon^2(t)} \tag{29}$$

where Φ and $\overline{\epsilon^2(t)}$ denote the infinite time averages of XX^T and $\epsilon^2(t)$. Then Equation (27) becomes

$$[I + 2kc\,\Phi]\frac{dw}{dt} = -k\nabla_w[\,\overline{\varepsilon^2(t)}\,] \,, \tag{30}$$

which is similar to the ideal control law in Equation (22), except for the extra term I on the left. To see how the weights behave under this equation, we collect all terms involving w on the left side. Since

$$\nabla_w[\,\overline{\varepsilon^2(t)}\,] = 2[\,\Phi w - S\,] \tag{31}$$

(see Equation (12a)), we have

$$[I + 2kc\,\Phi]\frac{dw}{dt} + 2k\Phi w = 2kS \,. \tag{32}$$

Now a typical weight has the solution

$$w_i(t) = A_{i_1} e^{-\frac{2k\lambda_1}{1+2kc\lambda_1}t} + A_{i_2} e^{-\frac{2k\lambda_2}{1+2kc\lambda_2}t} + \cdots + A_{i_{2M}} e^{-\frac{2k\lambda_{2M}}{1+2kc\lambda_{2M}}t} + c_i. \tag{33}$$

The j^{th} time constant in this transient response is

$$\tau_j = \frac{1+2kc\lambda_j}{2k\lambda_j} \,. \tag{34}$$

which may be compared with Equation (10) for the LMS algorithm. We find that now, as λ_j becomes large, τ_j does not become arbitrarily small as in Equation (10) but is bounded below by c. By choosing c properly, we may limit the fast response speed of the array without limiting the signal ratio I/N.

The steady-state solution of Equation (8), the LMS algorithm, is

$$w = \Phi^{-1} S, \tag{35}$$

which is known to be the optimal solution (1). We note, however, that Equation (32) has the same steady-state solution, regardless of the value of c, so it also yields optimum weights.

The difference between the LMS algorithm and Equation (32) is in their transient behavior. In particular, if c is large enough that

$$2kc\lambda_j > 1 \tag{36}$$

for <u>every</u> eigenvalue λ_j, Equation (32) may be approximated by

$$2k\Phi\left[c\frac{dw}{dt} + w\right] = 2kS\ , \tag{37}$$

or simply

$$c\frac{dw}{dt} + w = \Phi^{-1}S\ , \tag{38}$$

which is equivalent to the ideal control law in Equation (22).* In this case, all components w_i have the same time constant, c. There is only one time constant in the array response, and it is independent of signal power.

Equation (30) differs from the ideal control law because of the extra term I on the left. This difference means that when the array receives weak signals, so $2kc\Phi$ is negligible compared to I, Equation (30) becomes the LMS algorithm. With strong signals and large $2kc\Phi$, the I term is negligible and Equation (30) becomes the ideal control law. For in between cases, there will still be some spread in time constant if $2kc\lambda_j < 1$ for some λ_j. But with the loop modified as shown in Figure 3, the shortest time constant is dictated by c rather than by the strongest signal.

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*Substitute Equation (31) into Equation (22) to obtain this form.

An additional remark is helpful for understanding the control loop in Figure 3. If Equation (34) is compared with Equation (10), we see that each τ_j for the new feedback loop is larger than the corresponding τ_j for the LMS loop. Therefore, when the modifications shown in Figure 3 are added to the LMS loop, the result is to slow down the response time of the array. To obtain a fixed speed of response from the array, c is chosen large enough that the fast time constant terms in the LMS loop are slowed down until they are of the same speed as, or slower than, the slow terms in the LMS loop. The modified loop in Figure 3 is thus slower than the LMS loop, but has the advantage of fixed time constants. Constancy of speed of response is much more important for system design purposes than obtaining the fastest possible response. Of course, the real time speed of response of a hardware implementation would be adjusted to a suitable value by choosing gains appropriately.

We now return to the averaging operation $A\{\cdot\}$ in Equation (24). $A\{\cdot\}$ was defined as a finite time average over an interval short compared to changes in w_i and dw_i/dt, but long compared to the fluctuations of $x_i(t)$ and $R(t)$. We now elaborate on this definition.

Let us rewrite Equation (26) in the form

$$2k\ A\left\{x_i(t)\left[R(t) - \sum_{j=1}^{2M} x_j(t)w_j\right]\right\} = 2k\ A\{x_i(t)R(t)\} - 2k \sum_{j=1}^{2M} A\{x_i(t)x_j(t)\}\ w_j. \tag{39}$$

Using this and Equation (25) in Equation (24), we find that the weights in the modified loop satisfy the system

$$[\,I + 2kc\ A\{XX^T\}]\ \frac{dw}{dt} + 2k\ A\ \{XX^T\}\ w = 2k\ A\ \{XR(t)\}\ . \tag{40}$$

First, we observe that the matrix $A\{XX^T\}$ multiplying dw/dt must be nonsingular if the modified loop is to have the desired behavior. To see why this is so, assume for the moment that the matrix $A\{XX^T\}$ can be approximated by a constant matrix, and make a rotation of coordinates in Equation (40) into the principal axes of $A\{XX^T\}$. Let

$$w = B\eta \, , \tag{41}$$

where B is a 2M x 2M orthogonal coordinate rotation matrix and η is the weight vector expressed in the principal axes of $A\{XX^T\}$ (the "normal" weight vector). Let

$$\Lambda^* = B^T A\{XX^T\} B$$

$$= \begin{pmatrix} \lambda_1^* & 0 & \cdots & \\ 0 & \lambda_2^* & & \\ & & \cdot & \\ \cdot & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & & \lambda_{2M}^* \end{pmatrix} \tag{42}$$

be the matrix of eigenvalues of $A\{XX^T\}$. After rotating coordinates, we find that the j^{th} normal weight η_j satisifes the differential equation

$$(1+2kc\lambda_j^*) \frac{d\eta_j}{dt} + 2k\lambda_j^* \eta_j = 2kq_j, \tag{43}$$

where q_j is the j^{th} component of the column vector Q:

$$Q = B^T A\{XR(t)\} \, . \tag{44}$$

As long as $2kc\lambda_j^* > 1$, we see that the time constant for η_j will be approximately c. For any given set of λ_j^*, we can assure that all transients in the array response have time constant c by choosing c large enough that $2kc\lambda_j^* > 1$ for all λ_j^*, including the smallest one. Clearly it will be possible to do this only if the smallest eigenvalue is not zero.* If $A\{XX^T\}$ has any zero eigenvalues, the feedback loop modification in Figure 3 will not have the intended effect.

These remarks make it clear that some averaging is definitely necessary. For, without averaging,

$$A\{XX^T\} = XX^T, \tag{45}$$

and the matrix XX^T is always of rank 1. (X is an eigenvector of XX^T with eigenvalue X^TX. Any other vector orthogonal to X is an eigenvector with eigenvalue 0. Hence XX^T always has 2M-1 zero eigenvalues.) We should not expect fixed time constants in this case.**

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*$A\{XX^T\}$ is positive semidefinite, so all $\lambda_j^* \geq 0$.

**If the averaging is omitted, the weights satisfy the system obtained by substituting Equation (45) in Equation (40):

$$[I+2kcXX^T]\frac{dw}{dt} + 2kXX^Tw = 2k\,XR(t)$$

This may be rearranged by multiplying on the left by the inverse

$$[I+2kcXX^T]^{-1} = \left[I - \frac{2kc}{1+2kcX^TX}\,XX^T\right]$$

which yields

$$\frac{dw}{dt} + \left[\frac{2k}{1+2kcX^TX}\right]XX^Tw = \left[\frac{2k}{1+2kcX^TX}\right]XR(t)$$

Comparing this with Equation (4) for the LMS algorithm shows that the only difference between the two is that the gain constant 2k in the LMS loop is replaced by the quantity $\frac{2k}{1+2kcX^TX}$ in the modified loop. As long as $2kcX^TX > 1$, the effect of this change is simply to normalize the loop gain of the modified loop to X^TX, the total power in the array. This has the effect of fixing the fastest time constant in the array, but does not solve the problem of time constant spread.

Next, with $A\{\cdot\}$ defined as a finite time average, we must determine what averaging time is required to make all the eigenvalues nonzero. This question is most easily answered by considering the problem in discrete form. Suppose the signals in the array are sampled every ΔT seconds. Let X_j denote the j^{th} sample of vector X,

$$X_j = \begin{pmatrix} x_1(j\Delta T) \\ x_2(j\Delta T) \\ \vdots \\ x_{2M}(j\Delta t) \end{pmatrix} . \tag{46}$$

A finite time average of XX^T over an interval T can be approximated by an average of K samples of XX^T:

$$A\{XX^T\} = \frac{1}{T}\int_{t-T}^{t} XX^T dt \cong \frac{1}{K}\sum_{j=1}^{K} X_j X_j^T , \tag{47}$$

where $T = K\Delta T$. Clearly, at least 2M samples are required to make $A\{XX^T\}$ nonsingular. I.e., the matrix $X_1X_1^T$ is of rank 1, $1/2(X_1X_1^T + X_2X_2^T)$ is of rank 2 (if X_1 and X_2 are not collinear)*, $1/3(X_1X_1^T+X_2X_2^T+X_3X_3^T)$ is of rank 3 (if X_1, X_2 and X_3 are not coplanar), and so forth. Hence to be nonsingular, the matrix

$$\frac{1}{K}\sum_{j=1}^{K} X_j X_j^T \tag{48}$$

must contain at least 2M samples (i.e., $K \geq 2M$). For K=2M, the matrix will be nonsingular as long as the X_i are linearly independent.

To make the vectors X_i independent, it is sufficient that the sampling times for the X_i be far enough apart. Since

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*We can always find 2M-2 vectors orthogonal to both X_1 and X_2, so there will always be 2M-2 zero eigenvalues for $1/2(X_1X_1^T+X_2X_2^T)$.

$$X_i^T X_j = \sum_{k=1}^{2M} x_k(i\Delta T)x_k(j\Delta T), \tag{49}$$

we have

$$E\{X_i^T X_j\} = \sum_{k=1}^{2M} R_{x_k}[(j-i)\Delta T], \tag{50}$$

where $E\{\cdot\}$ denotes the expectation and $R_{x_k}(\tau)$ is the autocorrelation function of $x_k(t)$,

$$R_{x_k}(\tau) = E\{x_k(t)x_k(t+\tau)\}. \tag{51}$$

If the sampling time ΔT is large enough that all the terms $R_{x_k}[(j-i)\Delta T]$ are small, then $E\{X_i^T X_j\}$ is small, and X_i and X_j are nearly orthogonal, on the average. The value of ΔT required to make $R_{x_k}(\tau) \cong 0$ may be determined from the spectral density of the signals $x_k(t)$. For example, suppose the signals have flat power spectral density $S_x(\omega)$ of P_o watts/radian per second over a bandwidth of B Hz., as shown in Figure 4:

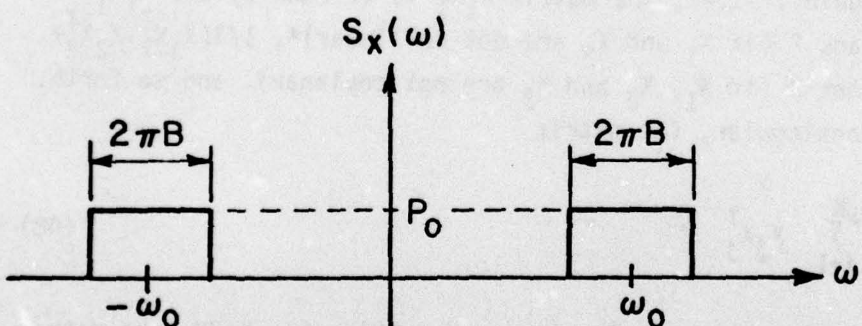

Figure 4. Power spectral density of $X_i(t)$.

Then the autocorrelation function (the Fourier Transform of $S_x(\omega)$) is

$$R_{x_k}(\tau) = 2P_0 B \frac{\sin \pi B \tau}{\pi B \tau} \cos \omega_0 \tau \, , \tag{52}$$

so we can insure that on the average the vectors x_i are orthogonal by choosing

$$\Delta T \leq \frac{1}{B} \, . \tag{53}$$

Since at least k=2M samples of X will be required to make

$$\frac{1}{K} \sum_{j=1}^{K} X_j X_j^T$$

nonsingular, the time interval used in Equation (47) should be approximately

$$T = \frac{2M}{B} \, , \tag{54}$$

i.e.,

$$A\{XX^T\} = \frac{B}{2M} \int_{t - \frac{2M}{B}}^{t} XX^T \, dt \, . \tag{55}$$

We may also express the required averaging time in terms of carrier cycles. Since the carrier period is

$$T_0 = \frac{1}{f_0} \, , \tag{56}$$

where f_0 is the carrier frequency in Hz., we have

$$T = \frac{2M}{(B/f_0)} \quad T_0 \, . \tag{57}$$

Note that B/f_0 is the fractional bandwidth.

In practice, simulations of Equation (24) indicate that the averaging time can be somewhat smaller than this amount. The reason is that to make $A\{XX^T\}$ nonsingular requires only linear independence of the X_i, not orthogonality. The value of ΔT given in Equation (53) makes the X_i orthogonal, which is a stronger condition. In general, one finds that as the averaging time is reduced, c must be made larger to maintain $2kc\lambda_j > 1$ for the smallest eigenvalue. The smallest eigenvalue goes to zero as the averaging time is reduced.

V. AN EXAMPLE

Now we give a simple example to illustrate the behavior of the feedback loop in Figure 1. Consider a two-element array of omnidirectional elements with four quadrature weights, as shown in Figure 5. We assume a CW desired signal of amplitude A_d is incident on the array from broadside at frequency ω_o. We also assume a double sideband, suppressed carrier AM interference signal of amplitude A_i, carrier frequency ω_o, and modulation frequency ω_m is incident from an angle θ_i off broadside. The resulting signals in the array are

$$\begin{aligned} x_1(t) &= A_d\cos\omega_o t + A_i\cos\omega_m t\,\cos(\omega_o t-\phi_i), \\ x_2(t) &= A_d\sin\omega_o t + A_i\cos\omega_m t\,\sin(\omega_o t-\phi_i), \qquad (58) \\ x_3(t) &= A_d\cos\omega_o t + A_i\cos\omega_m t\,\cos\omega_o t, \end{aligned}$$

and

$$x_4(t) = A_d\sin\omega_o t + A_i\cos\omega_m t\,\sin\omega_o t.$$

where

$$\phi_i = \frac{\omega_o L}{c}\sin\theta_i \; . \qquad (59)$$

L is the element spacing and c is the velocity of light.* We let the reference signal be

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*This c is unrelated to the gain constant c defined earlier.

$$R(t) = \cos\omega_o\tau \, . \tag{60}$$

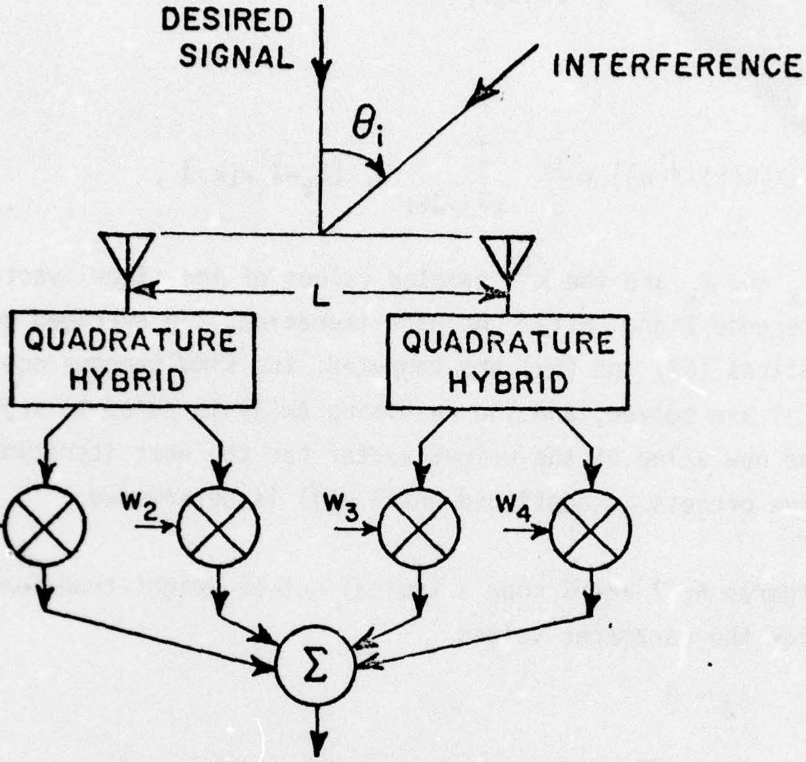

Figure 5. A two-element array.

The array weights satisfy Equation (40). To determine typical weight transients, we have solved these equations numerically using a discrete (difference equation) approximation, as follows. All quantities in the equation are sampled array ΔT seconds. We let w(i) be the i^{th} sampled value of the weight vector. We approximate

$$\frac{dw}{dt} \cong \frac{\Delta w(j)}{\Delta T} = \frac{w(j+1)-w(j)}{\Delta T} \quad . \tag{61}$$

Substituting this in Equation (40) yields an equation for $\Delta w(j)$:

$$[I + 2kcA_j\{XX^T\}] \ \Delta w(j) = 2k\Delta T A_j \ \{X[R(t)-X^T w]\} \, , \tag{62}$$

where $A_j\{\cdot\}$ denotes the average of the quantity in brackets at the j^{th} sample. $A_j\{\cdot\}$ is computed as a moving average over the last J samples; i.e.,

$$A_j\{XX^T\} = \frac{1}{J} \sum_{k=j-J+1}^{j} X_k X_k^T \ , \qquad (63)$$

and

$$A_j\{X[R(t)-X^T w]\} = \frac{1}{J} \sum_{k=j-J+1}^{j} X_k \ [R_k - X_k^T w(k)] \ , \qquad (64)$$

where X_k and R_k are the k^{th} sampled values of the signal vector X and the Reference Signal R(t). At each iteration, the averaged quantities in Equations (63) and (64) are computed, the simultaneous equations for $\Delta w(j)$ are solved, and the resulting $\Delta w(j)$ is added to w(j) to produce the new value of the weight vector for the next iteration. This iterative process is continued until w(j) is determined.

Figures 6, 7 and 8 show a typical set of weight transients, computed for the parameter values

$A_d = 3$

$2k = .05$

$c = 100$

$\Delta T = \frac{\pi}{2\omega_o}$ (4 samples/carrier cycle)

$\frac{\omega_o L}{c} = \pi$(half-wavelength element spacing)

$\theta_i = 30^o$

$\omega_m = 0.5\omega_d$

and $A\{\cdot\}$ is an average of 8 samples (J=8).*

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*Note that according to Equation (57), the averaging time should be 4 carrier cycles or 16 samples, since we sample 4 times per carrier cycle. However, simulation results indicate that an 8 sample average is adequate in this case.

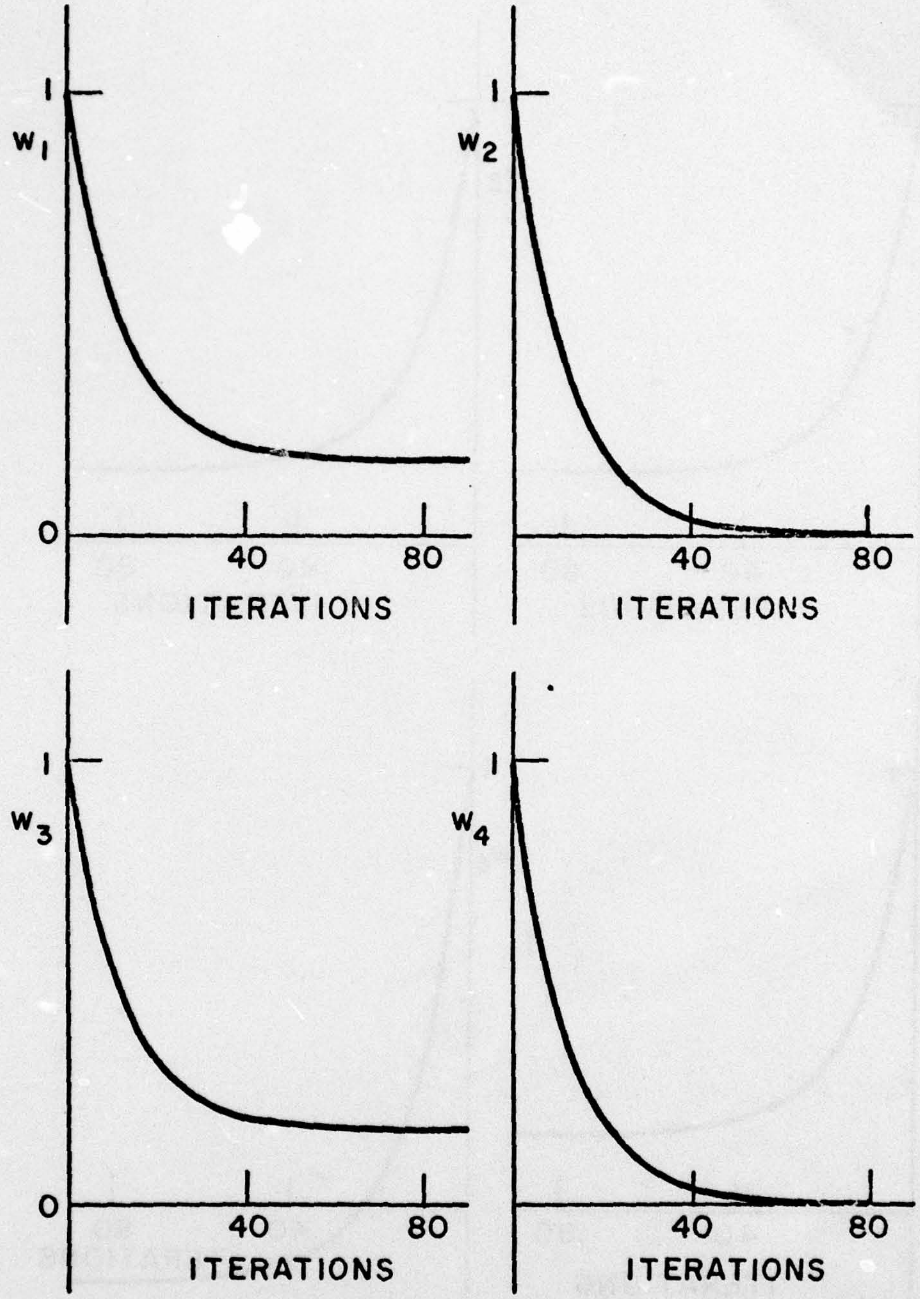

Figure 6. Weight transients with no interference.

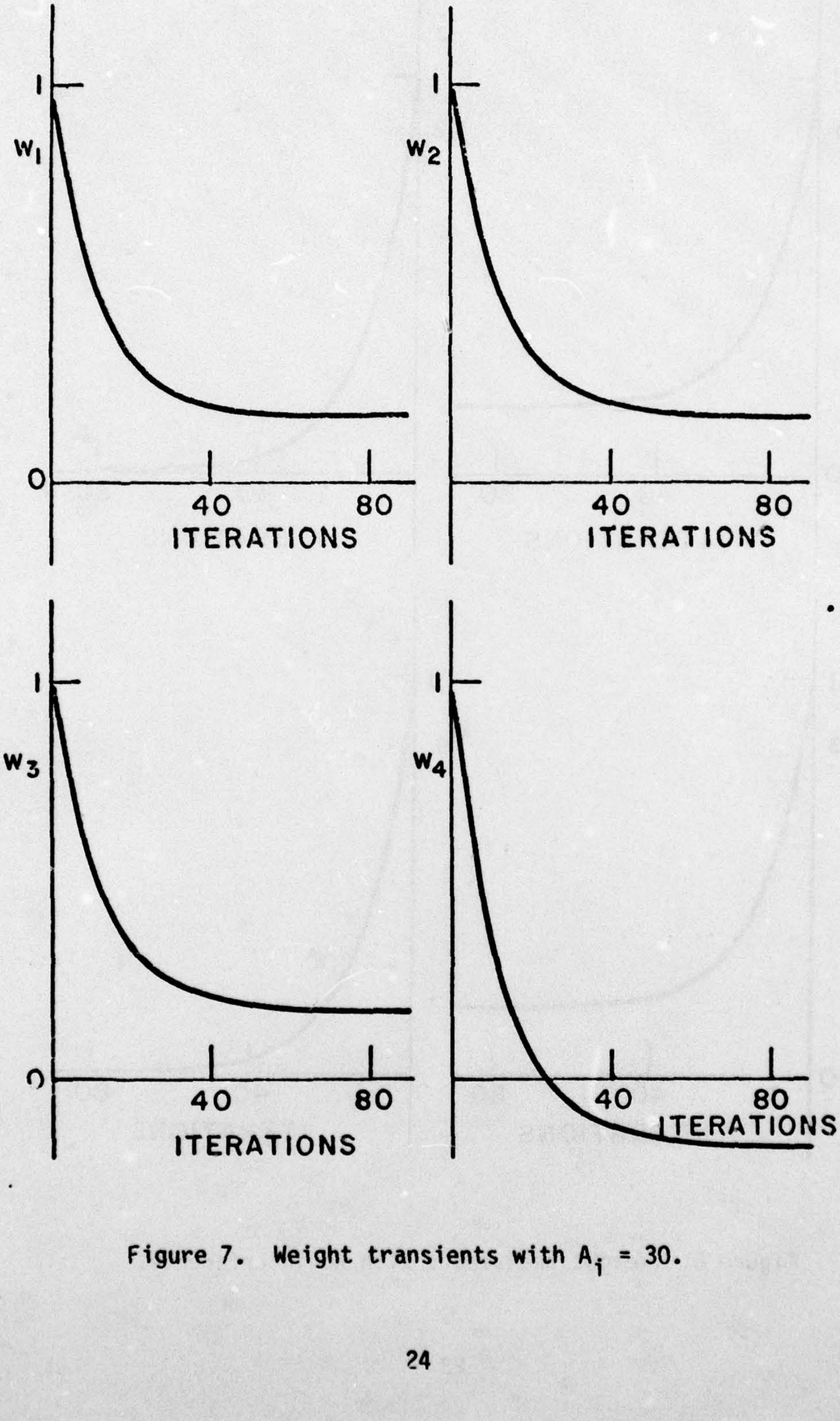

Figure 7. Weight transients with A_i = 30.

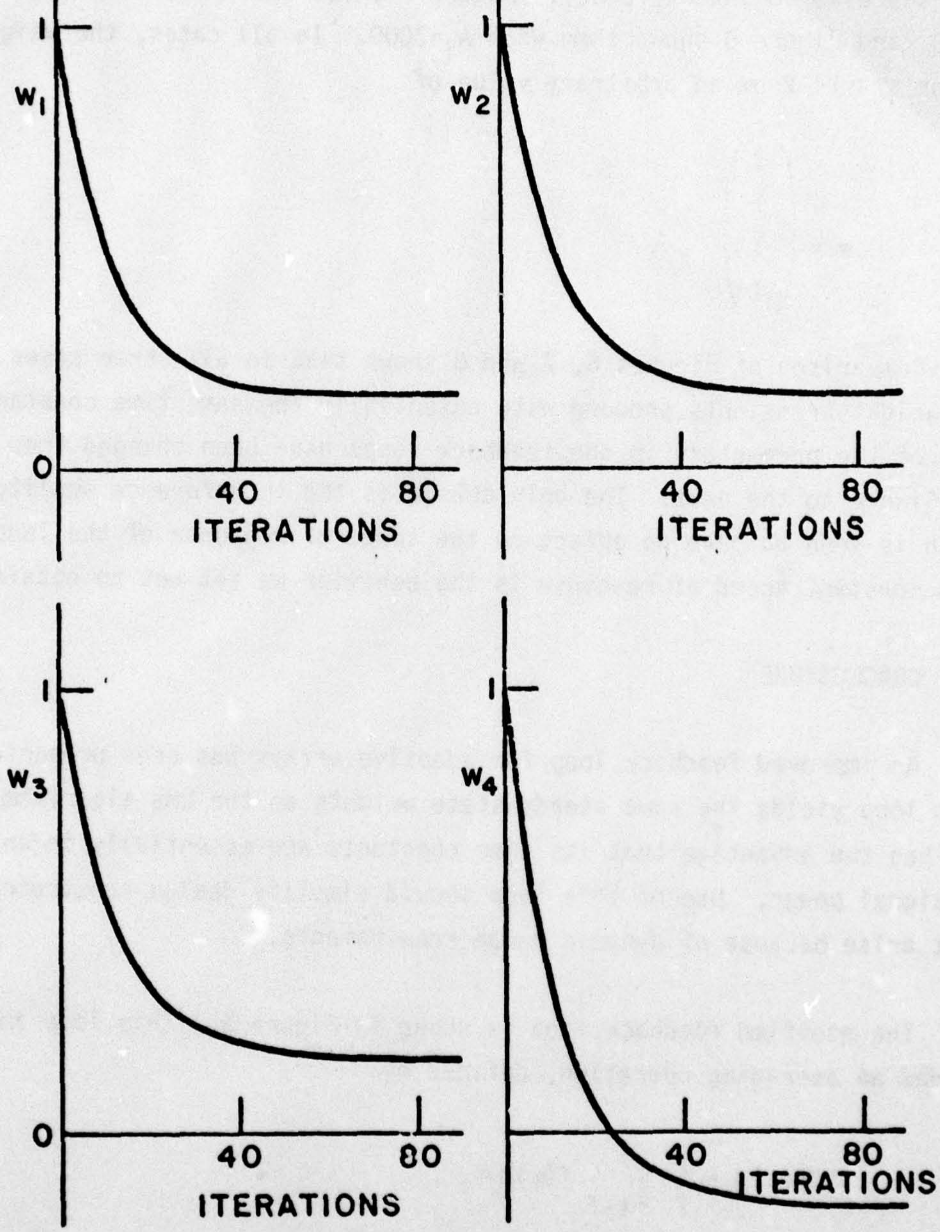

Figure 8. Weightytransients with A_1 = 3000.

Figure 6 shows the weight transients that result when $A_i=0$ (i.e., when there is no interference). Figure 7 shows the transients when $A_i=30$, and Figure 8 shows them when $A_i=3000$. In all cases, the weight vector starts from an arbitrary value of

$$w = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} .$$

Comparison of Figures 6, 7 and 8 shows that in all three cases the weight transients proceed with essentially the same time constant. None of the parameters in the feedback loops have been changed from one figure to the next. The only change is the interference amplitude, which is seen to have no effect on the speed of response of the loops. This constant speed of response is the behavior we set out to obtain.

VI. CONCLUSIONS

An improved feedback loop for adaptive arrays has been presented. This loop yields the same steady-state weights as the LMS algorithm, but has the advantage that its time constants are essentially indendent of signal power. Use of this loop should simplify design constraints that arise because of dynamic range requirements.

The modified feedback loop is shown in Figure 3. This loop includes an averaging operation, defined by

$$A\{f(t)\} = \frac{1}{T} \int_{t-T}^{t} f(\eta)d\eta .$$

An averaging time of approximately

$$T = \frac{2M}{(B/f_0)} \quad T_0$$

is adequate to make the system have the desired behavior, where M is the number of array elements, B/f_o is the fractional bandwidth, and T_o is the carrier period.

As a closing remark we comment that the feedback modifications shown in Figure 3 can also be used with an array of the type originally described by Applebaum (5). In this case, the reference signal in Figure 3 is eliminated and the main beam direction is controlled by adding an appropriate steering vector component to each weight. This type of array is useful when the desired signal angle of arrival is known in advance.

REFERENCES

1. B. Widrow, P. E. Mantey, L. J. Griffiths and B. B. Goode, "Adaptive Antenna Systems," Proc. IEEE, 55, 12 (December 1967), 2143.

2. R. T. Compton, Jr., R. J. Huff, W. G. Swarner and A. A. Ksienski, "Adaptive Arrays for Communication Systems: An Overview of Research at The Ohio State University," Trans. IEEE, AP-24, 5 (September 1976), 599.

3. R. T. Compton, Jr., "An Adaptive Array in a Spread Spectrum Communication System," Proc. IEEE, 56, 3 (March 1978), 289.

4. K. L. Reinhard, "Adaptive Antenna Arrays for Coded Communication Systems," Technical Report 3364-2, October 1973, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract F30602-72-C-0162 for Rome Air Development Center. RADC-TR-74-102 AD 782395.

5. S. P. Applebaum, "Adaptive Arrays," Trans. IEEE, AP-24, 5 (September 1976), 585.